Chinese Physics B

Evolution of IPv6 Internet Topology with Unusual Sudden Changes*

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(Received 05-30-2012; revised manuscript received 12-10-2012)

The evolution of Internet topology is not always smooth but sometimes with unusual sudden changes. Consequently, identifying patterns of unusual topology evolution is critical for Internet topology modeling and simulation. We analyze IPv6 Internet topology evolution in IP-level graph to demonstrate how it changes in uncommon ways to restructure the Internet. After evaluating changes of average degree, average path length and some other metrics over time, we find that in the case of a large-scale growing the Internet becomes more robust; whereas in a top-bottom connection enhancement the Internet maintains the same efficiency with a simplified core.

Keywords: scale-free network, Internet topology evolution, unusual evolution of Internet topology, complex network analysis.

PACS: 89.20.Hh, 89.75.Hc, 89.75.-k, 05.20.-y

1. Introduction

The Internet has been evolving rapidly over time like a living organism, so has its topology[1]. Since any experiment on the Internet is unacceptably expensive, understanding mechanism of Internet topology evolution becomes an important topic for related research. It provides an essential knowledge to examine the existing infrastructure and protocols of the Internet, and

*Research supported by the National Natural Science Foundation of China (Grant No. 60973022).

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also helps scientists in the research of modeling Internet to do experiments without potential damages. Moreover, it helps us to comprehend the objective laws behind such a complex network so that scientists can understand, model, predict and control complex systems.

As a typical example of complex network, Internet topology research also includes four objectives like other network science research. They are 1) topology detection, 2) topology characteristic analysis and mechanism discovery, 3) network modeling and 4) application of a discovered mechanism or a proposed model. Such objectives of Internet topology exploration not only glean from network science research but also make great contributions to the development of the discipline.

Thus, significant contributions have been given in the field. Among continuous works on probing topology[2, 3], modeling[4, 5, 6, 7], analyzing properties[8, 9, 10, 11, 12], constructing generators[13], understanding community structure[14, 15] and developing complex network theory[16, 17], many efforts focus on statistics or graph theory. For instance, Wang et al.[18] demonstrated scale-free statistics of BBS Servers Visiting, showing that power-law-like characteristic exists in a certain real-world complex system. Yuan et al.[19] presented a model with exponential cut-off degree distribution and local-cluster property. Li Ying et al.[20] analyzed the principle of the Internet topology evolving and gave a conjecture on the optimal structure of the Internet, also based on the statistical characteristics.

However, to the best of our knowledge, most of these interesting works focus on the normally evolving topology of complex network. In other words, how a complex network topology normally looks like has been answered again and again. In our case, general patterns of Internet topology evolution have been overwhelmingly explored with unusual sudden changes ignored. On the other hand, the IPv4 Internet topology is so huge that some unusual changes of a partial topology are often covered by the normal majority. Therefore, it is our objective to identify unusual sudden changes in the IPv6 Internet topology and demonstrate the reasons of the changes. Our work is based on four-month CAIDA (The Cooperative Association for Internet Data Analysis) detection data-set for IPv6 Internet topology.

The rest of this paper is organized as follows. In section 2, necessary definitions are cited or devised. Then, considering some important Internet topology metrics, we trace topology evolution over time and identify unusual evolutions based on the proposed definition. In section 3, the details of Internet topology are examined to illustrate reasons of the sudden changes under the surface. In section 4, we discuss the discoveries and make the conclusions.

2. Normal and Unusual Evolution of Internet Topology: Definitions and Observations

In a IP-level graph, considering each IP address as a node and a connection between two nodes as a link, we have a data-set of Internet topology as a graph G, which contains a set of nodes V and a set of links L. With the aim to distinguish unusual changes of metrics from normal evolution, we given the following definition.

Definition 1 *For any chosen metric M of Internet topology evolving over time, let m_i ($i \in \{1, 2, \cdots, n\}$) represent rate of change over time sequence t_i. $R_i(m_i) = [m_i - \epsilon_i, m_i + \epsilon_i]$ is a neighborhood of m_i, where ϵ_i is a stochastic variable. Denote $R_i + R_j = R_i \cup R_j$, $\forall i, j \in \{1, 2, \cdots, n\}$, we construct a sequence of region $F_1(m_1) = R_1(m_1)$, $F_2(m_2) = R_2(m_2)$ and $F_i(m_i) = R_{i-1}(m_{i-1}) + R_{i-2}(m_{i-2}), i \in \{3, 4, \cdots, n\}$. Internet topology evolves normally for metric M at time t_i, if $m_i \in F_i(m_i)$. It unusually evolves for metric M at time t_i if $m_i \notin F_i(m_i)$. We call $F_i(m_i), i \in \{1, 2, \cdots, n\}$, as Fibonacci evolution region sequence.*

By Definition 1, whether a value in a sequence is unusual or not depends on its two previous values and a stochastic variable ϵ_i. For example, if there is a metric that stayed around 10 for many time units, at certain point A_1 it drops to 3. Statistically, A_1 is a outlier and can be easily isolated. But if at the very next time unit A_2, the value is still 3. In the case of $A_2 = 3$, it is also a outlier according to statistics. Until the average of sample is low enough, 3 is a unusual value of such sequence. However, it is a quite normal point by Definition 1 since $A_2 \in R(A_1)$. On the other hand, if there is a sample metric that stays around 10 for 100 time units, then drops to 3 at B_1 and stays at 3 for another 100 time units. Statistically, 10 or 3 is normal due to the distribution of the whole sample. But the time unit that the metric drops from 10 to 3 is really unusual and need to be discussed. Therefore, with the ϵ_i decided by traditional statistical method, Definition 1 rules out the normal case $A_2 = 3$ in our first example and picks out the unusual case $B_1 = 3$ in the second one, which is very helpful for our analysis of unusual changes of metric during evolution.

Definition 2 [21] *Consider an unweighed graph G with a set of nodes V. Let $d(v_1, v_2)$, where v_1 and $v_2 \in V$ denote the shortest distance between v_1 and v_2. Assume that $d(v_1 = v_2) = 0$ if $v_1 = v_2$ or v_2 cannot be reached from v_1. Then the average path length (average distance) l_G is:*

$$l_G = \frac{1}{n \cdot (n-1)} \cdot \sum_{i,j} d(v_i, v_j), \tag{1}$$

where n is the number of nodes in G.

Definition 3 [22] *A k-core of a graph G is a maximal connected subgraph of G in which all nodes have degree at least k. Equivalently, it is one of the connected components of the subgraph of G formed by repeatedly deleting all nodes of degree less than k. If a non-empty k-core exists, then clearly, G has degeneracy at least k, and the degeneracy of G is the largest k for which G has a k-core. Moreover, a node u has coreness c if it belongs to a c-core but not to any (c+1)-core.*

Definition 4 *$V_1, V_2, \cdots, V_i, \cdots, V_n$ are node data-sets for each time unit. V_i is node data set in time unit i, where $i \in [1, 60]$. For series Internet topology data-sets over time, ratio of remaining nodes (R_i), ratio of emerged nodes (E_i) and ratio of missing nodes (M_i) are defined as*

$$R_i = \frac{V_i \cap V_{i-1}}{V_i} \times 100\% \tag{2}$$

$$E_i = \frac{V_i - (V_i \cap V_{i-1})}{V_i} \times 100\% \tag{3}$$

$$M_i = \frac{V_{i-1} - (V_i \cap V_{i-1})}{V_{i-1}} \times 100\% \tag{4}$$

We can deduce similar concepts for link data-sets in the Internet topology.

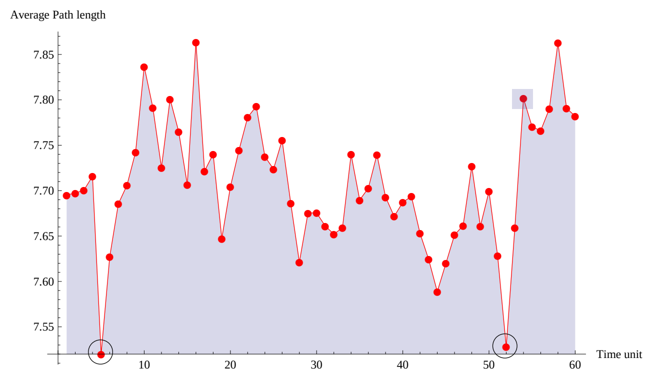


Figure 1: Average path length of Internet topology

Definition 5 *$L_1, L_2, \cdots, L_i, \cdots, L_n$ are link data-sets for each time unit. L_i is link data-set in time unit i, where $i \in [1, 60]$. For series Internet topology data-sets over time, ratio of remaining edges (R_i'), ratio of emerged edges (E_i') and ratio of missing edges (M_i') are defined as*

$$R_i' = \frac{L_i \cap L_{i-1}}{L_i} \times 100\% \tag{5}$$

$$E_i' = \frac{L_i - (L_i \cap L_{i-1})}{L_i} \times 100\% \tag{6}$$

$$M_i' = \frac{L_{i-1} - (L_i \cap L_{i-1})}{L_{i-1}} \times 100\% \tag{7}$$

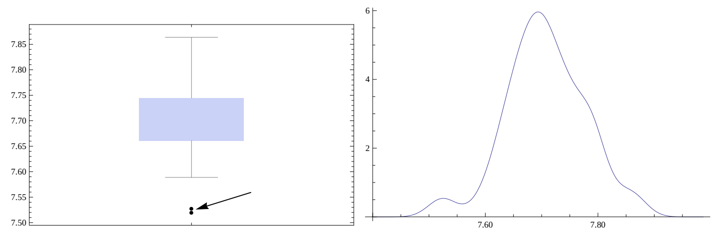
The reason we given Def. 4 and 5 is to evaluate the Internet topology evolution considering change rate of nodes and edges, which in some cases is critical to show the unusual time unit such as time unit 30 in Fig.11.

In the following paragraphs, several metrics of Internet topology evolution are calculated. We mainly focus on two of the three robust measures of network topology: clustering coefficient and average path length [24]. And average degree, number of nodes and number of links are also concerned.

2.1. Observation on Average Path Length

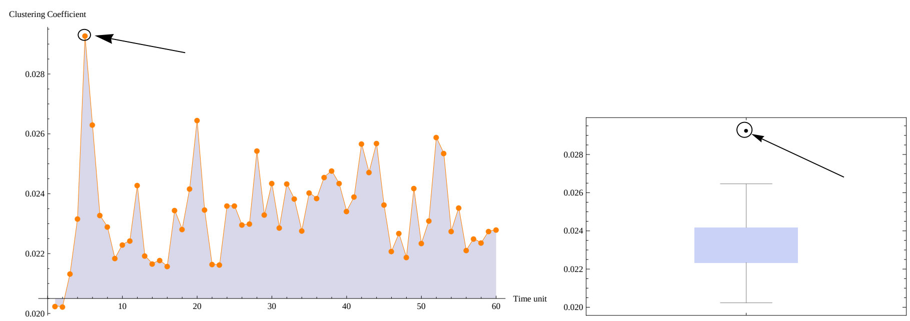
Because a short average path length (APL) facilitates the quick transfer of information and reduces time costs in the Internet topology, it is applied to measure the transmission performance of Internet topology. In Fig. 1, x-axis indicates time units which is $i = 1 \ldots 60$.

By Definition 1, we have two unusual evolution units for average path length (APL) when ϵ=0.05. They are labeled with black circles in Fig. 1. Comparing with Fig. 2, it is clear that



(a) Box-and-whisker chart for Average path length (b) Smooth histogram of Average path length

Figure 2: Statistical visualization for average path length



(a) Chart for Clustering Coefficient (b) Box-and-whisker chart for Clustering Coefficient

Figure 3: Charts for Clustering Coefficient

these two units is out of normal region of APL. The value at time unit 5 and 52 is far too low from the average.

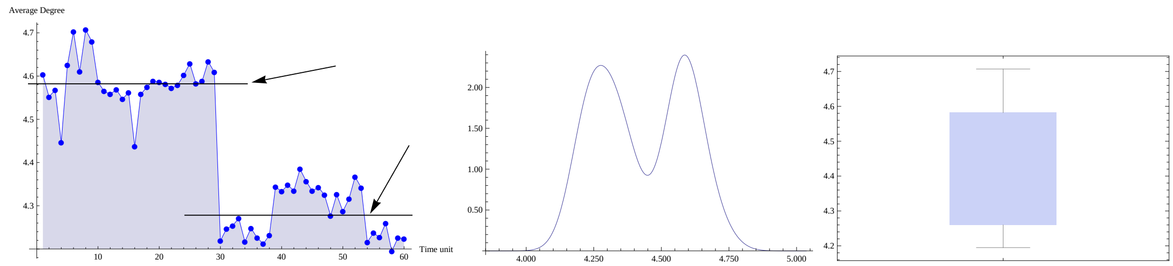
2.2. Observation on Clustering Coefficient

Clustering coefficient of Internet topology indicates the measure of degree to which nodes of Internet topology tend to cluster together.

Shown in Fig. 3(a) and 3(b), clustering coefficient is very near the average value except at time unit 5 ($i = 5$). It is corresponding to the observation of APL at time unit 5. At time unit 52, clustering coefficient goes slightly up but still in the normal region.

2.3. Observation on Average Degree

Average degree shows the density of the overall Internet topology. The time-series trend of average degree is in Figure 4. Values of average degree is on axis-y against time units on axis-x.



(a) Plot for Average Degree (b) Smooth histogram (c) Box-and-whisker Chart

Figure 4: Charts for Average Degree

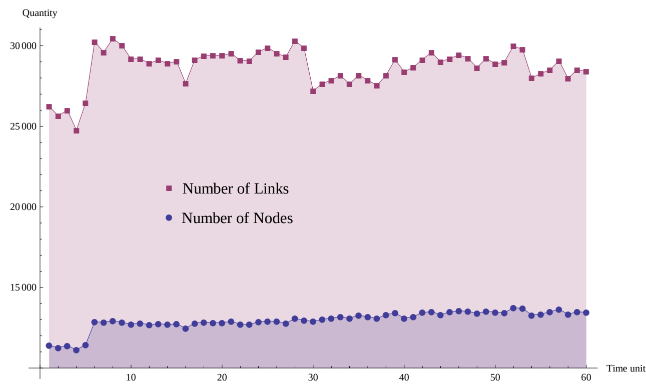


Figure 5: Number of nodes and links of Internet topology in time series. Red squares indicate link quantity and Blue dots represent node quantity.

After comparing Fig. 4(a) with Fig. 4(b), we can clearly see that average degree has a twin-peak distribution where at time unit 30 ($i = 30$) average degree goes directly down. It decreased to about 4.2, before that day the average is 4.56. So according to Definition 1 for average degree it is another unusual evolution of Internet topology in time unit 30.

2.4. Observation on Nodes and Links

Nodes and links are basic elements of Internet topology. All the internal changes of Internet topology eventually show up as changes of nodes and edges. It is an important factor we need to consider.

Shown in Fig. 5, quantity of nodes in Internet topology has an increasing tendency and takes a leap at $i = 6$ but not at $i = 5$ like the unusual evolution of APL. By contrast, quantity of links significantly drops at $i = 30$, with quantity of nodes continuing its increasing, which is corresponding to the unusual evolution of average degree.

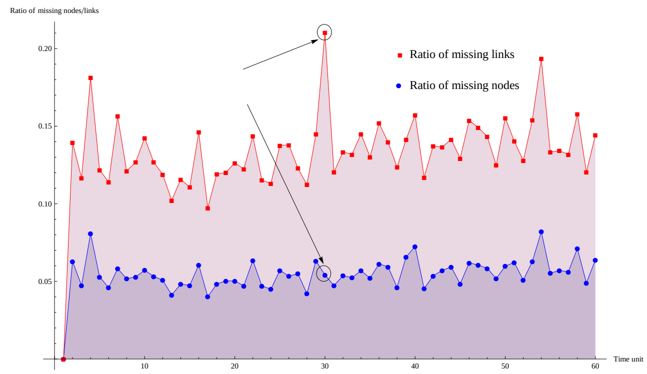


Figure 6: Ratio of missing nodes M_i (blue dot) and ratio of missing links M_i' (red square) over time

Therefore, by our observations, the unusual evolutions of Internet topology clearly exist during a four-month time period. We see that the metrics of Internet topology change suddenly in an obvious unusual way at some time units. Which part does Internet topology evolve causing these sudden changes? We need to search the structure of Internet topology for details.

3. Under the Surface: Reasons of Sudden Changes

In this section, we search the reasons make Internet topology evolve unusually. Base on our analysis, one unusual decrease of links at $i = 30$ and one unusual increase of links and nodes at $i = 5$ were selected for further exploration. A conversion of degree distribution in Internet topology was found, explaining the unusual evolution at $i = 30$. Sub-network connection of Internet was discovered, explaining the unusual increase at $i = 5$.

3.1. The Change of Nodes and Links

We trace nodes and links over time, by classifying nodes into three types. 1) There are remaining old nodes, 2) newly emerged nodes and 3) nodes will disappear in next time unit. It is noticeable missing nodes in next time unit are included in the remaining and emerged nodes in this time unit. By the Definition 4 and 5, the results of tracing these three kinds of nodes are shown in Fig. 6, 7 and 8.

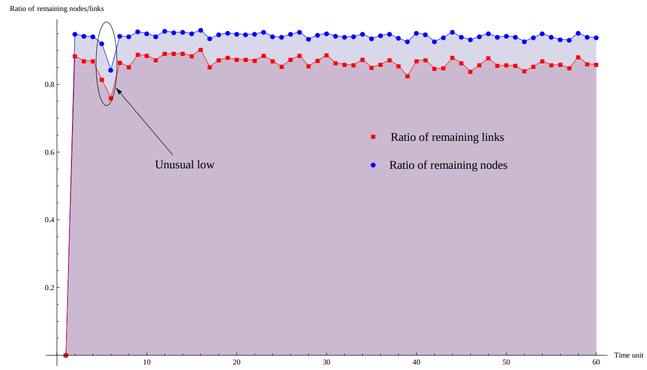


Figure 7: Ratio of remaining nodes R_i (blue dot) and ratio of remaining links R_i' (red square) over time

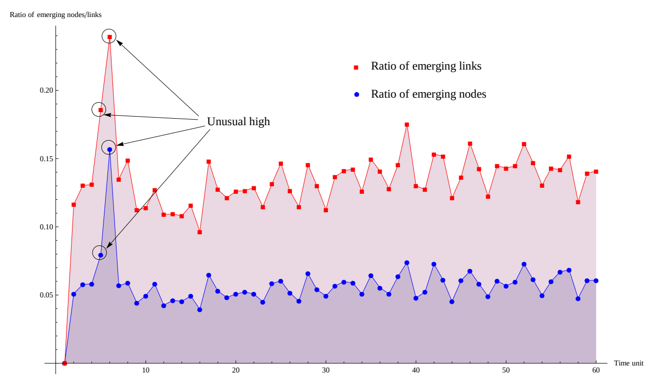


Figure 8: Ratio of emerged nodes N_i (blue dot) and ratio of emerged links N_i' (red square) over time

In Fig. 7 and 8, only at $i = 5$ and $i = 6$, ratio of the emerged nodes/edges and ratio of the remaining nodes/edges answer the Definition 1 of unusual evolution. The number of nodes and the number of links have a large increase at $i = 5$ and $i = 6$. Meanwhile, ratios of missing nodes and links are normal (i.e., Fig. 6). By Definition 4 and 5, we can deduce that a large amount of nodes and links emerge at $i = 5$ and $i = 6$ without any increase of the ratio of missing nodes and links. Therefore, a large group of new nodes connect to the Internet IPv6 network in the period (i.e., $i = 5$ and $i = 6$). Those new nodes and links are detected for the first time in the topology, which means the Internet IPv6 topology grows.

In Fig. 9, red big nodes are newly emerged at $i = 5$. The distribution of the nodes shows everywhere within topology. Table 1 confirms the result showing most k-cores have new emerged nodes at $i = 5$ and most of the nodes are in small k-cores. Since we know at $i = 5$ the Internet topology evolve unusually and the reason for this change is that topology grows 8%

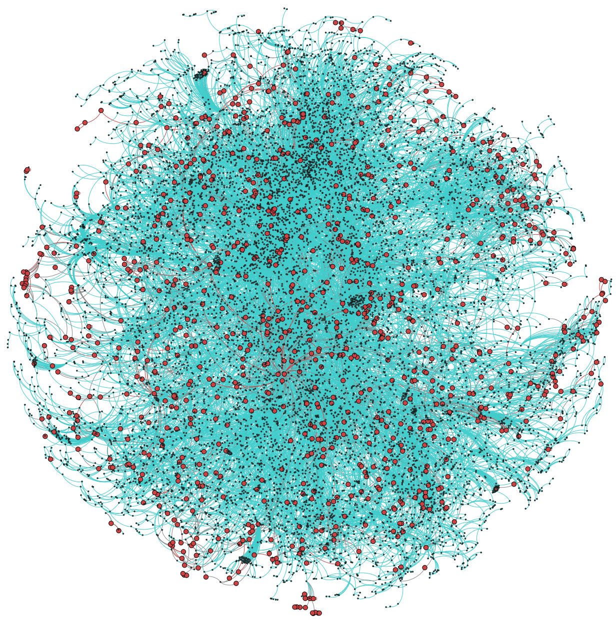


Figure 9: Newly emerged nodes at $i = 5$

Table 1: Distribution of new emerged nodes in different k-cores

K-core	Number of new emerged nodes
1	196
2	505
3	133
4	38
5	11
6	8
7	2
8	10
9	2
10	1
11	2

larger at $i = 5$ and 15% larger at $i = 6$, it shows that sudden large-scale growing of Internet topology can cause an unusual evolution for certain metrics. We call this pattern of Internet evolution "growing pains of Internet topology". Moreover, Fig.10 shows the difference of degree distribution between $i = 4$ and $i = 5$. Blue circles are the node degree distribution at

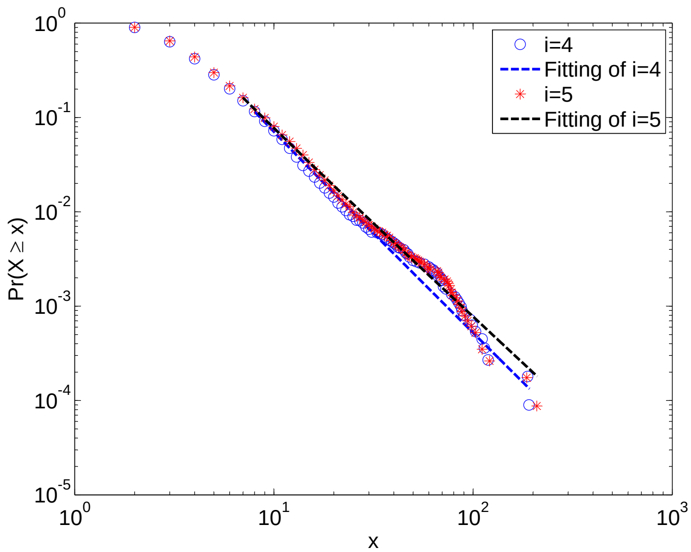


Figure 10: Degree Distribution Comparison between $i = 4$ and $i = 5$ in log-log coordinate

$i = 4$, while the blue dash line indicates its fitting straight line in log-log coordinate. On the other hand, red stars show the node degree distribution at $i = 5$ with the higher black dash line standing for its fitting. We use the method proposed by Clauset et al.[23] to measure the power-law characteristic. Results infer that time unit $i = 5$ and $i = 6$ have the same degree distribution, but at $i = 5$, the number of medium degree nodes relatively increases, i.e. there are more nodes with degree between 7 and 30.

At time unit $i = 5$ and $i = 6$, Internet topology grows bigger in large scale. The APL has a minimum value at $i = 5$ whereas clustering coefficient stay unusually higher than the average. According to Albert's paper [24], the robustness of topology is enhanced by the growing of the Internet.

3.2. The Conversion of Degree Distribution

On the other hand, it is clear ratio of missing links is uncommonly high at $i = 30$, but ratio of missing nodes at the same time unit is perfectly normal (Fig. 7). It means number of missing nodes is normal in $i = 30$, but more links are missing compared to the previous time units. As Fig. 11 shows, if there is one missing node at any time unit i, there are usually 2 to 2.4 missing links. The ratio of missing links to missing nodes is an approximate constant except at $i = 30$. The question is: which links disappear at time unit $i = 30$?

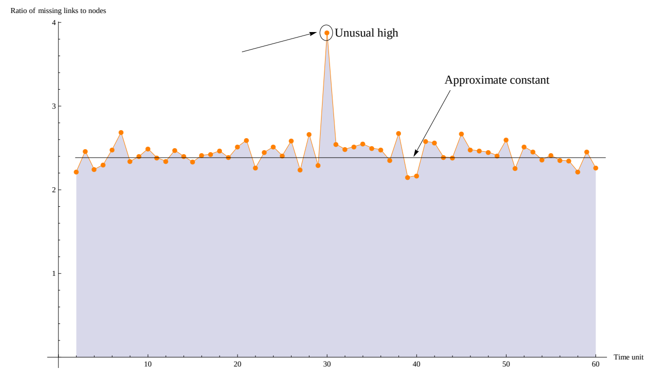
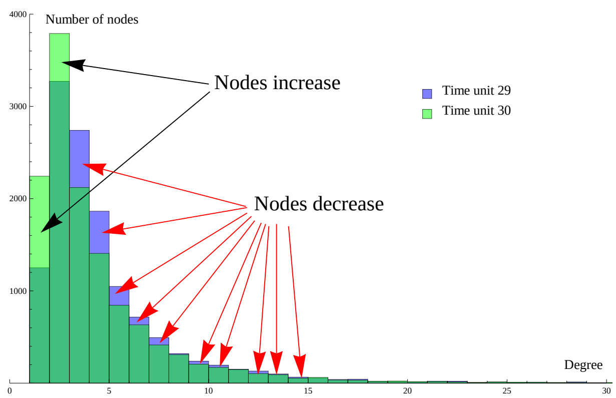
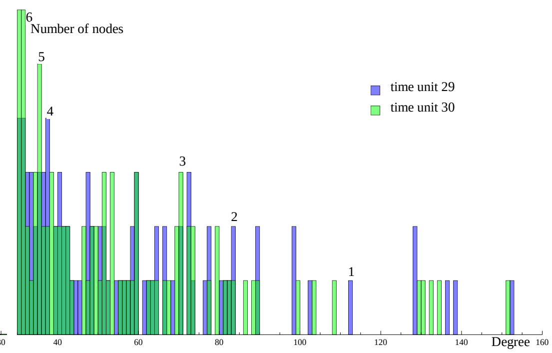


Figure 11: Ratio of missing links to missing nodes ($\frac{M_i'}{M_i} \times 100\%$)



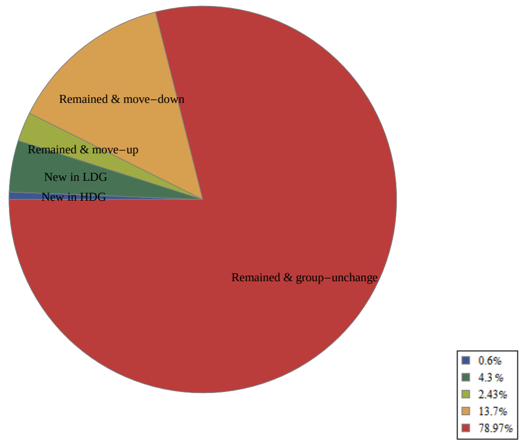
(a) Degree Distribution from 1 to 30



(b) Degree Distribution from 30 to 160

Figure 12: The comparison of degree distributions between $i = 29$ and $i = 30$. The blue stands for missing nodes in $i = 30$ compared to $i = 29$, green is the shared nodes between $i = 29$ and $i = 30$. And the light green indicates the emerged nodes in $i = 30$. Degree value of nodes is on axis x and quantity of node is on axis y.

In Fig. 12, the comparison of degree distributions between $i = 29$ and $i = 30$ shows that the lower degree group of nodes, with degree of 1 or 2, increase its quantity. Meanwhile the higher degree group of nodes, with degree larger than or equal to 3, decrease its number. We know that the quantity of nodes remains steady near the average in this time unit. So there are two possible reasons for the change in Fig. 12. 1) Many new nodes emerge in the lower degree group while old nodes disappear in the higher degree group. 2) Many old nodes lose their links, their degrees become smaller and they move from higher degree group to lower degree group. As Figure 6 and 7 show, the ratio of missing nodes and ratio of emerged nodes is quite normal at $i = 30$ compared to previous time units. It means there is no large quantity of nodes missing or emerged. Thus, at $i = 30$, many nodes with high degree decrease their degree, move to the



(a) (b)

Figure 13: Composition of nodes at time unit 30

lower level and become low-degree nodes. This conversion of nodes moving from higher degree group to lower degree group explains the missing links. Besides, with degree distributions compared in all time units, this moving of large amount of nodes from higher degree group to lower degree group is found only at $i = 30$.

Moreover, since degree-one and degree-two nodes in Fig. 12 obviously increased and almost all other higher degree levels have a decrease in the quantity of nodes. To simplify the layers and structure of Internet topology in our following comparison, k-cores by the Definition 3 is used. It is because the scope of degrees of Internet topology is from 1 to more than 150 (Fig. 12). Meanwhile scope of k-cores is no more than 11 in our case.

Before we decompose the k-cores of Internet topology at $i = 29$ and $i = 30$, one other thing need to be clarified first. Nodes at $i = 30$ are divided into two classes: low degree group (LDG) with degree one and two; high degree group (HDG) with degree larger than 3 (including 3). They are both relative concepts, corresponding to classification of nodes in Fig. 12. If a node was in HDG but moves down into LDG, we call it a move-down node. On the other hand, if a node was in LDG but moves up into HDG, we term it as a move-up node. If a node was in HDG/LDG and remains in the same group in the next time unit. We call it a group-unchanged node.

In Fig. 13, the blue part and deep green part in the pie chart are the newly emerged nodes at $i = 30$. The difference is the blue part emerges in HDG but the deep green part emerges in LDG. However, the blue is smaller then the deep green. As a whole, the impact of them makes

Table 2: Distribution of remaining degree-decrease nodes at time unit 30

K-Core	$i = 29$	$i = 30$	Changes
1	1	924	+923
2	544	844	+300
3	1015	0	-1015
4	200	0	-200
5	8	0	-8

Table 3: Distribution of newly emerged nodes at time unit 30

K-Core	$i = 29$	$i = 30$
1	0	244
2	0	357
3	0	30
4	0	4
5	0	1

the average degree decrease. On the other hand, the light green part in pie chart indicates the move-up nodes in $i = 30$, It is only 2.43 percents of all, not large enough to make the average degree appear higher. The brown is the move-down nodes and it's 13.7 percents of all. Except the group-unchanged nodes, the brown has the most of number. Therefore, it has the largest impact on the average degree of Internet topology. Together, the brown and deep green nodes make the average degree drop. They are 18% of all nodes.

Table 2 shows that k-cores at $i = 30$ has a distribution such that the remaining degree-decrease nodes move from high k-cores to low k-cores. These nodes are the brown part of pie chart in Figure 13. And in Table 3 we can see newly emerged nodes at $i = 30$ mostly appear only in k-core 1 and 2 (more than 90%). The result is corresponding to our previous observation. In addition, it is noticeable these changed nodes are all in k-core 1 to 5. The absence of the high k-core nodes in this conversion indicates high degree nodes are slightly affected by the unusual evolution at $i = 30$.

Next, we show the link distribution of those remaining degree-decrease nodes at $i = 30$ compared to $i = 29$ in Table 4.

Links in LDG increases due to the conversion. By contrast, links in HDG decrease dra-

Table 4: Distribution of links belonging to remaining degree-decreased nodes

Class of Links	$i = 29$	$i = 30$	Changes
Links within LDG	0	536	536
Links between LDG and HDG	747	2175	1428
Links within HDG	3979	0	-3979

Table 5: Distribution of remaining degree-increase nodes in $i = 40$ compared to $i = 39$

K-Core	$i = 39$	$i = 40$	Changes
1	1481	1475	-6
2	4561	4417	-144
3	2337	2390	53
4	524	564	40
5	62	114	52
6	17	16	-1
7	0	5	5
10	0	1	1

matically. In particular, links between the top and bottom groups increase the number almost three times than the ones within LDG. In other words, this conversion of Internet topology does not only make nodes in high degree group move to low degree group but also highly increase the number of links between HDG and LDG. It is a top-bottom connection enhancement in the Internet.

In Figure 4(a), we can see average degree goes back at $i = 40$. Table 5 shows the k-core distribution of remaining degree-increase nodes in $i = 40$. Though there are some nodes move back from low degree group to high degree group, the number is rather small compared to the conversion at $i = 30$. Yet we track all node changes in every time unit, the conversion is confirmed irreversible. Most of the move-down nodes at $i = 30$ never move back.

At $i = 30$, Internet topology evolves normally for average path length and clustering coefficient, but average degree unusually changes. Further exploration tells us, with the number of nodes unchanged and number of links highly decreased, the Internet topology can still maintain a normal APL and clustering coefficient. The degree distribution and link distribution do change remarkably.

Since this link redistribution at $i = 30$ is found right after the Egypt revolution happened. And the links and nodes are moved from higher layers to lower layers in the topology, just like the rights of Egypt were given back to common people. We name this unusual evolution of Internet topology as "revolution phenomenon". During the process, APL basically stayed the same and clustering coefficient slightly changed, as well as diameter of network. With in-neglectable link decrease, Internet still maintains its efficiency. This result is consistent with the conclusion Zhang et al.[25] presented, which prove that deleting "black sheep" edges from high betweenness nodes can enhance the transmission efficiency of Internet. Moreover, our findings (Table.4) indicate, if the connection between the top rulers (HDG) and bottom civilians (LDG) can be enhanced, the Internet can maintain its efficiency with a much simplified core.

4. Conclusion

In this paper, our main contributions are the observations of the unusual evolution of Internet topology and the reasons of such sudden changes. We also give a definition to distinguish normal evolution from unusual evolution. By presenting the evolution of average degree, number of nodes and links, etc., we find two patterns of this kind of unusual evolution that researchers usually ignore.

When the Internet topology has a large-scale growing, it may cause a unusual evolution with APL suddenly decreases while clustering coefficient increases. When the Internet topology has an inside link redistribution, average degree would drop and ratio of missing linking could get extremely high. Meanwhile the APL shows nothing unusual.

As a conclusion, even if most of the metrics of Internet remain unchanged, the Internet topology may has a huge restructure inside. And even if a lot of links of Internet topology are missing at certain point, the effectiveness of Internet topology transmission can still be maintained.

Acknowledgment

The research is supported by the National Science Foundation of China (No.60973022).

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