TAKE BOARDS!

Ian A. McCulloh[1], Howard D. McInvale[2], and Richard Gussenhoven[3]

ADDRESS: Department of Mathematical Sciences, United States Military Academy, West Point, New York 10996 USA. (1) ai6873@usma.edu, (2) ah2499@usma.edu, and (3) ar3820@usma.edu.

ABSTRACT: The practice of students going to the boards to work problems has been a tradition in undergraduate education for over two hundred years. A study was conducted to determine what instructional techniques enabled students to better learn fundamental concepts in mathematics. This study identifies board work as a highly effective instructional technique for developing a student's ability to succeed on a standardized exam covering basic algebra, geometry, and trigonometry skills. This study offers insight into student motivation, and it suggests that board work improves student performance and learning.

KEYWORDS: Mathematics, blackboard, fundamental concepts, recitation, instructional techniques, regression, West Point, statistics.

The practice of students going to the boards to work problems has been a tradition in undergraduate education for over two hundred years [1, p.2]. A study was conducted to ascertain the efficacy of board work as a way to help students better learn fundamental concepts in mathematics. This paper describes a study of the 2004 freshmen mathematics course at the United States Military Academy, and demonstrates that it is a tradition worth keeping.

Board work at the Academy is a teaching tool that is as old as the Academy itself. Perhaps no one method has so influenced the quality of instruction as that of blackboard recitations. Use of the blackboard was first introduced at West Point by George Baron, a civilian teacher. In 1801, Baron gave Joseph Swift, West Point's first graduate, "a specimen of his mode of teaching at the blackboard."[4, p. 25] In March of 1817, Claude Crozet, a West Point professor and graduate of the Polytechnic Institute of Paris, used the chalkboard as a way of instructing the first descriptive geometry course at the Academy. In July of that year, Major Sylvanus Thayer was appointed Superintendent of the Academy and expanded the use of board work. The freer use of board work evolved into more effective recitations where students actually solved problems at the black boards. [3, p. 42] To this day, students at the Academy do work at the board to showcase their understanding of mathematical concepts.

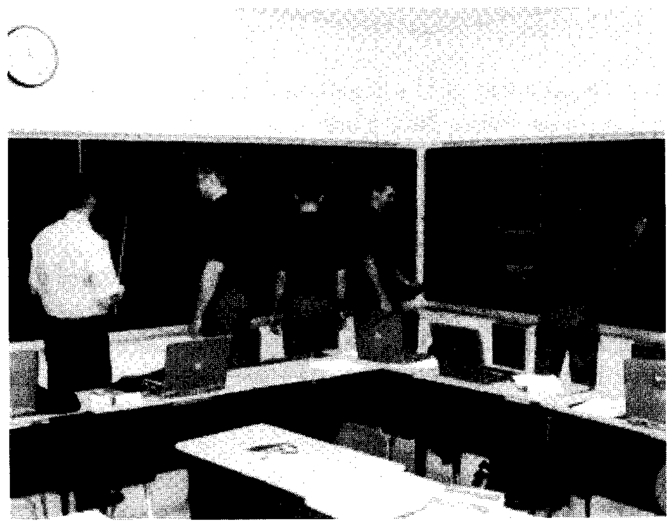


Figure 1. Cadets working at boards in modern times.

Oral recitation was a widespread method of mathematics instruction in the 19^{th} century. In fact, prior to 1881, even examinations in mathematics were almost entirely oral. [3, p. 91] By the end of the 19^{th} century, the use of the blackboard to aid in recitation was found to be a highly effective tool at West Point. Colonel O.H. Ernst, the Superintendent of the Academy, describes this tradition in his 1896 Annual Report—

> "[The cadet goes to the board], writes his name on the upper-right hand corner of it, and proceeds to place upon it the formulas, equations, and intermediate mathematical work necessary for a clear and complete demonstration...of the subject. [As soon as the cadet is finished his work], he takes a pointer in his hand, faces the instructor, and stands at attention until called upon to recite....As the cadet finishes his recitation, the instructor [evaluates the work and the cadet returns to his seat]."[3, p. 77]

The Superintendent introduced written examples and exercises to the instruction process in 1881, because he believed that demonstrations were essential. Professor A. E. Church, the head of the Mathematics Department, subsequently began to use both oral recitations and written examinations. The Superintendent reported the success of this approach to the Adjutant-General of the U. S. Army in his Annual Report by stating,

> "The result has been extremely satisfactory, and I believe that the present system of combining written recitations and examinations with the oral compares favorably with that employed in any other similar institution. I am convinced that the successful students acquire a better understanding of the principles than formerly, and the percentage of failures has of late years diminished." [3, p. 91]

Our study of the 2004 freshmen mathematics course at the Academy suggests that the Superintendent, Colonel Ernst, was right indeed, and the use of board work results in a diminished percentage of failures on an examination.

In the past, the Academy placed a great deal of emphasis on the standardization of instruction. Instructors were expected to follow the same lesson plan for each class session, a lesson plan usually dictated by the course or program director. In recent years, however, the Academy has utilized several initiatives to develop discipline-specific pedagogy. Two of these initiatives are the USMA Center for Teaching Excellence (CTE), established in 1994, and the Master Teacher Program, launched in 2002. The result of these initiatives is an ongoing discourse about teaching methods and techniques, with instructors seeking various methods to promote student learning. Thus, instructors in the Department of Mathematical Sciences differ in their approaches to preparing students for standardized exams. This study identifies board work as the most effective instructional technique for developing a student's ability to succeed on one such standardized exam, the Fundamental Concepts Exam (FCE) (Appendix 1).

The FCE is administered during the first mathematics course that students take at the Academy. This exam assesses a student's knowledge of basic algebra, geometry, and trigonometry skills traditionally taught in high school, and it validates student proficiency in these areas. Over the past decade, the exam has evolved from a mechanically-oriented skills exam, to one with a more conceptual flavor. Following the FCE, in the Fall of 2004, freshmen mathematics instructors were surveyed about the instructional techniques they used to prepare students for the exam. The survey (Appendix 2) identifies seven different instructional techniques used by 21 instructors for over 800 students. Instructional techniques ranged from formal algebra, geometry, and trigonometry instruction to no instruction in these fundamental concepts. The seven instructional techniques are explained below:

1. *Instructors presented formal instruction in fundamental concepts.* Although fundamental concepts are not specified teaching objectives, a few instructors took time to formally present certain key concepts.
2. *Students did FCE type problems in class at the boards.* Some instructors required students to work problems on the chalk board in class, similar to traditional board recitations.
3. *Students given an FCE quiz.* Some instructors gave their sections quizzes on fundamental algebra, geometry, and trigonometry concepts very similar to the FCE within the first five weeks of the semester.
4. *Instructors verified whether students did practice problems.* All students were given additional, FCE style, suggested practice problems. Some instructors collected or verified in some way that the students were completing these suggested problems.
5. *Students given a difficult non-FCE quiz with a class average below 50%.* Some instructors gave a difficult quiz, early in the semester, with a resulting class average of less than 50%. It was hypothesized that this motivated students to study harder for the FCE.
6. *Instructors incorporated informal instruction of fundamental concepts.* When working more complex problems, instructors pointed out embedded fundamental concepts as they were used.
7. *Instructors emphasized the negative consequences of failing the FCE.* Instructors emphasized in varying degrees the consequences of failing to achieve the 80% standard on the FCE.

The efficacy of these instructional techniques was assessed using multiple linear regression where the above techniques were predictors. The following four responses were investigated.

- *Raw FCE Performance.* Raw FCE Performance is the student's FCE exam score.
- *FCE Improvement.* Student entrance exams cover FCE type material. FCE Improvement is the difference in score between a student's entrance exam and their FCE.

- *Raw FCE Performance (Bottom 25%).* Raw FCE Performance (Bottom 25%) is the FCE score for students among the lowest scoring 25% on the entrance exam.
- *FCE Improvement (Bottom 25%).* FCE Improvement (Bottom 25%) is the difference in score between the entrance exam and the FCE for students among the lowest scoring 25% on the entrance exam.

All seven instructional techniques were investigated for their effect on each of the four responses. Each technique was tested for its significance of regression with a t-test. Out of the seven techniques, only one was found to have a statistically significant effect– *Students did FCE type problems in class at the boards.* Table 1 illustrates the impact of this instructional technique on each of the four responses.

Students Did FCE TypeProblems in Class at the Boards.		
Response	**t-statistic**	**P-value**
Raw FCE Performance	2.40	0.017
FCE Improvement	2.71	0.007
Raw FCE Performance(Bottom 25%)	2.61	0.011
FCE Improvement (Bottom 25%)	3.02	0.003

Table 1. Significance of Board Work as an Instructional Technique.
[Note: A statistically significant result (95% confidence) is one that has a P-value less than 0.05.]

Table 1 demonstrates that board work affects all four responses, with the greatest impact among the lowest performing students. It is interesting to note that board work was most effective for students among the lowest scoring 25% on the entrance exam. Thus, board work is a valuable tool for all students, but most valuable for weaker students, because the interaction between the student and instructor fosters an active learning process. As students showcase their knowledge at the boards, instructors evaluate their abilities. Their interactions allow students to demonstrate their understanding in front of their instructors who can then provide them direct feedback and resolve any points of confusion. This direct feedback enhances the understanding of fundamental concepts and improves student performance on the evaluation of these concepts.

The results of this study also offer insight into student motivation. Sending students to the boards forces them to practice solving problems, which aids in their learning. It can also motivate them to prepare for class, for fear of being embarrassed in front of their peers. Practice and repetition, coupled with tapping into a student's motivation, is well known to improve test performance. This study demonstrates that the Academy's long standing tradition of "going to the boards" is an effective means of motivating students to prepare and practice.

Clearly, the purpose of teaching is not to train students to do well on a test. Rather, the purpose of a test is to assess learning. Simply training students to pass the FCE does not serve their best interests in the long run. However, understanding of certain fundamental concepts is an expected prerequisite for success in the core mathematics, science, and engineering courses at the Academy. Therefore, as opportunities to integrate fundamental concepts avail themselves, teachers are encouraged to incorporate these skills into instruction that includes board work.

The results of this study generate two interesting questions. The first question is, "Does board work improve student knowledge of fundamental concepts?" The Academy uses national tests, such as the Scholastic Aptitude Test (SAT) and the American College Test (ACT), to assess a student's ability to succeed in undergraduate mathematics. West Point students typically perform well above the national average for mathematics. The average mathematics SAT (MSAT) score for students entering West Point in 2004 was 642 out of 800; while, the average MSAT score for college bound seniors nation-wide in 2004 was 518 out of 800. [5, p. 1] However, 46% of students entering West Point failed to achieve the desired 80% standard on a placement exam covering FCE material. Furthermore, on the FCE conducted during week five, 16% of the students still failed to pass with an 80%. In subsequent attempts to pass the FCE, the failure rate was 6% and 1% respectively. Did the students improve their knowledge of mathematics, or did they merely learn to recognize a particular series of problems in order to pass the FCE? Since they met the minimum mathematics requirements for entry into the Academy, does that not imply that they understand those basic entry-level mathematics concepts? To adequately assess the issue of retaining mathematics knowledge, a broader study is required. Such a study should track a sample of students throughout their undergraduate experience using periodic testing of FCE knowledge.

The second interesting question generated by this study is, "How does the evolution of technology impact the use of traditional board work?" The Academy has a long history of technological innovation in the classroom. Claude Crozet and other professors at the Academy in the 1820's were among the first professors in the nation to use the blackboard as the primary tool of instruction. In 1944, the slide rule was required for all students and was used in all freshmen mathematics classes. During William Bessell's tenure as head of the Department of Mathematical Sciences (1947-1959), the mathematics classrooms were modernized with overhead projectors and mechanical computers. Bessell was also instrumental in establishing a computer center at the Academy. The use of the hand-held calculator was mandatory for all West Point students beginning in 1975, and by 1986 all students had pre-configured computers. [2].

Today, all West Point students are required to buy state-of-the-art laptop computers with wireless internet capabilities and powerful mathematical software packages to help them learn. These technologies are wonderfully effective at allowing the machines to do more calculating, so that the students can do more thinking. Nevertheless, one insight gleaned from the FCE study is that there is still a place in the classroom for the proven instructional technique of board work. The good news is that board work and technology are not mutually exclusive. The landscape of mathematical education is grand enough to accommodate both methods of instruction. The combination of these strategies may achieve the best overall results. In light of this conclusion, the recommendation to combine the two methods harkens to the days of Colonel O.H. Ernst. If he were writing a 2004 USMA Annual Report, he might advocate "combining the two methods," and be convinced that "students [will] acquire a better understanding of the principles and the number of failures [will] diminish." However, one traditional colloquialism will have to be amended. Rather than telling students to "Take Boards," instructors will also have to tell them to "Take Laptops!"

REFERENCES

1. Anderson, Charnel. 1961. *Technology in American Education, 1650-1900.* Washington: U.S. Department of Health, Education, and Welfare.

2. Arney, David C. 1998. *Technological Development.* `http://www.dean.usma.edu/departments/math/about/history/tech.htm`. (31 Jan. 2005).

3. Ernst, O. H. 1896. *Annual Report of the Superintendent, USMA, 1896-1911.* Washington: Government Printing Office.

4. Richardson, Robert C. 1917. *West Point: An Intimate Picture of the National Military Academy and of the Life of a Cadet.* New York and London: G. P. Putnam's Sons.

5. College Entrance Examination Board. 2004. *2004 College-Bound Seniors: A Profile of SAT Program Test Takers (2004).* `http://www.collegeboard.com/prod_downloads/about /news_info/cbsenior/yr2004/pdf/2004_TOTALGRP_PRD.pdf` (27 Jun. 2004).

BIOGRAPHICAL SKETCHES

Ian A. McCulloh is an instructor in the Department of Mathematical Sciences at the U. S. Military Academy at West Point. He holds a BS in Industrial Engineering from the University of Washington ('94) and MS degrees in Industrial Engineering and Applied Statistics from the Florida State University ('04). Ian also serves as a research analyst in the Center for Data Analysis and Statistics at West Point. His research interests include applied statistics, regression, and response surface optimization.

Howard D. McInvale is an assistant professor and Course Director for MA 103 in the Department of Mathematical Sciences at the U. S. Military Academy at West Point. He has taught courses in discrete dynamical systems, mathematical modeling and probability and statistics. He holds a BS in American Politics from the U. S. Military Academy ('93) and an MS in Industrial and Systems Engineering from Virginia Polytechnic Institute and State University ('02). Doug is an infantry officer in the U. S. Army and serves as an analyst in the Operations Research and Systems Analysis career field.

Richard B. Gussenhoven is an instructor in the Department of Mathematical Sciences at the U. S. Military Academy at West Point. He has taught courses in single variable calculus and mathematical modeling. He holds a BS in Mechanical Engineering from the U. S. Military Academy ('94), an MS in Engineering Management from the University of Missouri-Rolla ('98) and an MS in Civil Engineering from the University of Massachusetts-Amherst ('04). Richard also serves as the Assistant Director for the Center for Data Analysis and Statistics at West Point.

APPENDIX 1
2004 FUNDAMENTAL CONCEPTS EXAM (FCE)

MA103 Fundamental Concepts Exam (FCE),
Term 05-1, version [A] 15 SEP 2004

Name ____________________ **Section**__________

Time Allotted: 55 minutes. This exam evaluates the understanding of the math concepts fundamental to each cadet at this stage of his/her academic development. This is a non-technology exam. No references of any kind may be used.

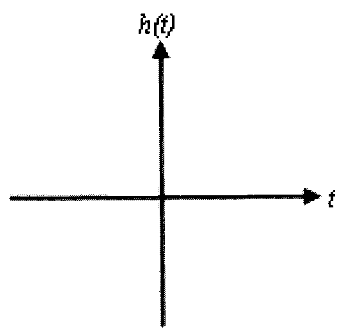
1. Given the function, $f(x) = x^2 + 3$, for what value(s) in the domain does $f(x) = 7$?

2. What is the slope of the line: $2y - 3x = 4$?

3. Do the following lines intersect? If so, how many times and where?

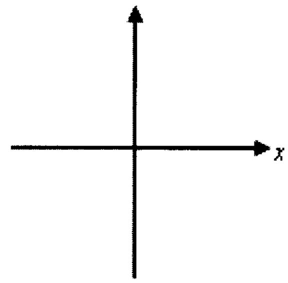
   Line 1: $x + 5y = 20$
   Line 2: $2y - 3x = -9$

4. Sketch the graph of the function $h(t) = \frac{1}{t}$ Identify three points on the graph.

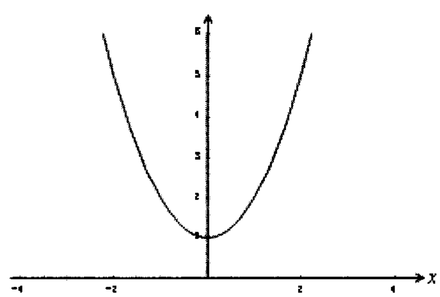


5. Determine the set of values in the domain and range of the function $g(x) = \sqrt{x+1}$.

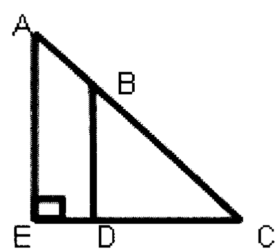
6. Sketch the graph of the equation: $2x + y = 4$.



7. Simplify the expression: $\frac{e^{2t}}{e^{4t-3}}$.
8. Given the function shown on the graph, estimate the value in the domain of this function that yields a function value of 2.
9. Let $t(u) = \frac{u^2}{2} + 3u$, what is $t(4)$?
10. What is an equation for the function shown below?



11. In the diagram below, side AE has a length of 4, side EC has a length of 3, and side DB has a length of 2. AE and DB are parallel. What is the length of side BC?



12. Find the area of the triangle whose vertices are at the following points: (1,2), (8,2), and (1,6)

13. The following is an equation for a circle:

$$(x-3)^2+(y+1)^2=4$$

Identify a point that lies on the circle and prove that your point is on the circle.

14. Give an example of a function whose domain is all real numbers, and whose value approaches zero as the independent variable gets very large.

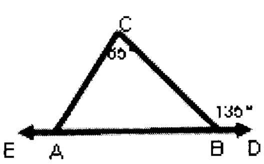
15. What is the equation for a line that is parallel to the y-axis and passes through the point (3,5)?

16. Given the function$g(s) = \sin(2s+17)$, what is the largest value $g(s)$ can have?

17. What happens to the function $h(v) = \frac{1}{v^3+1}$ as v gets close to -1?

18. Where does the graph of the function $f(x) = x^2 - 7x + 12$ cross the x axis?

19. In the diagram below, what is the angle EAC?



20. A line is drawn through the points $(1,1)$ and $(3,5)$. Another line is drawn through points $(-2,3)$ to $(0,6)$. Do these lines intersect? Why?

APPENDIX 2 – 2004 FCE

We are studying on the effectiveness of certain instructional techniques on cadet performance. The focus of the study is student performance on the Fundamental Concepts Exam (FCE). This will be a correlation study, considering instructional techniques as experimental factors. The responses will be raw FCE performance, FCE improvement (from summer placement exam), Raw FCE performance for students among the bottom 25% on the entrance exam, and the FCE improvement for the bottom 25%. The sole objective of the study is to identify if there are any instructional practices that make a significant difference in student FCE success.

Please take a moment to answer the following questions:

1. What sections do you teach:________________

2. Did you give cadets a quiz on FCE? Yes No

3. Did you give cadets a quiz (not FCE) with a class average below 50%
   Yes No

4. How many times did you collect/check/verify cadets did FCE problems? __________

5. Approximately how many times per week did you have cadets do FCE board problems?

   0 1 2 3 4 5

6. How many times did you formally teach fundamental concepts?

   0 1 2 3 4 5

   times per week or ___ per semester.

7. How many times did you informally teach fundamental concepts?

   0 1 2 3 4 5

   times per week or ___ per semester.

8. What kind of doomsday picture did you paint for consequences of failing the FCE?

Won't Graduate	Won't Pass MA103	Won't Do Well In MA103	Make-ups Will Be Offered